Fig. 2. Characteristic impedances.

relative shape of this rectangle which, in turn, determines whether the inner and outer conductors of the two coaxial systems will be close together or far apart. As $k \to 0$, side $AB \to \infty$, and both characteristic impedances $\to \infty$; while as $k \to 1$, side $0A \to \infty$ and both characteristic impedances $\to 0$. For the curves of Fig. 2, k varies from 0.005 to 0.65.

REFERENCES

[1] F. Bowman, "Notes on two-dimensional electric field problems," Proc. London Math. Soc., ser. 2, vol. 41, pp. 271–277, 1935.

[2] Arthur Cayley, An Elementary Treatise on Elliptic Functions. New York: Dover, 1961, pp. 179–188.

Comments on "Transmission Line Impedance Matching Using the Smith Chart"

P. I. SOMLO

For the recently discussed problem of R. M. Arnold[1] of finding the characteristic impedance and electrical length of a uniform, loss-free transmission line which transforms a given impedance into another given impedance, an alternate approach using an elementary auxiliary calculation (with no trial and error) has been given in [1].

REFERENCES

[1] P. I. Somlo, "A logarithmic transmission line chart," IEEE Trans. Microwave Theory Tech. (Corresp.), vol. MTT-8, p. 463, July 1960.

[1] R. M. Arnold, IEEE Trans. Microwave Theory Tech., vol. MTT-22. pp. 977–978, Nov. 1974.

An Alternate Derivation and Procedure for Cristal's Transformation for Transmission-Line Networks

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Abstract—This letter gives an alternate derivation and procedure for Cristal's transformation which transforms commensurate transmission-line networks with unit elements from the frequency variable λ to $1/\lambda$, where $\lambda = \tanh(\gamma L)$.

Given a network with input impedance $Z(\lambda)$ composed of commensurate transmission-line elements including unit elements, we will now give a procedure for finding a network with input impedance $Z(1/\lambda)$ directly from the element values of the given circuit. This derivation is believed to be simpler, the method easier to apply and somewhat more general.

The method is based on the following theorem: If the network of Fig. 1(a) has input impedance $Z(\lambda)$ then the input impedance of the circuit of Fig. 1(b) is $Z(1/\lambda)$.

Proof: The input impedance of the network of Fig. 1(a) is given by

$$Z_A = Z_0 \frac{Z_L(\lambda) + \lambda Z_0}{Z_0 + \lambda Z_L(\lambda)}. \tag{1a}$$

Since the unit element in Fig. 1(b) is terminated by a load impedance $Z_0^2/Z_L(1/\lambda)$, the input impedance of the network of Fig. 1(b) is given by

$$Z_B = Z_0 \frac{Z_0^2/Z_L(1/\lambda) + \lambda Z_0}{Z_0 + \lambda[Z_0^2/Z_L(1/\lambda)]} = Z_0 \frac{Z_L(1/\lambda) + (1/\lambda)Z_0}{Z_0 + (1/\lambda)Z_L(1/\lambda)}. \tag{1b}$$

Comparison of (1a) and (1b) shows that $Z_B = Z_A(1/\lambda)$.   Q.E.D. Using the above theorem we can easily transform ladder networks containing unit elements from the λ to the $1/\lambda$ plane. Consider as an example the network of Fig. 2(a) with input impedance $Z(\lambda)$. Using the above theorem we obtain the network of Fig. 2(b) with input impedance $Z(1/\lambda)$. Eliminating the impedance inverter then
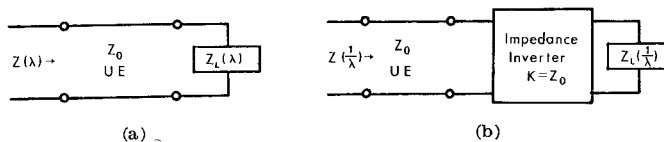
Fig. 1.   (a) A unit element terminated in a load impedance. (b) The transform of (a).
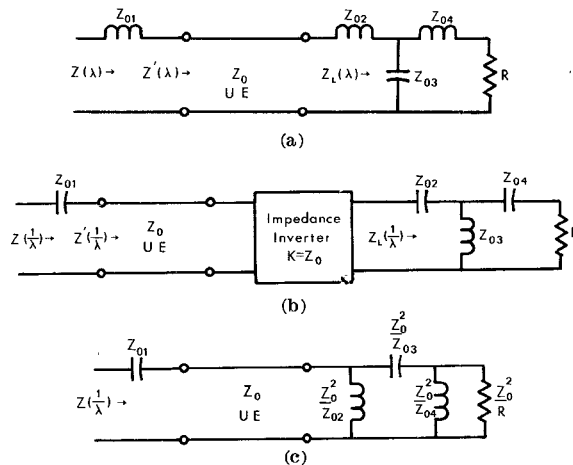

Fig. 2.   (a) A transmission-line network composed of inductive and capacitive stubs and a unit element. (b) The transform of (a). (c) Equivalent circuit of the circuit of (b) without impedance inverters.
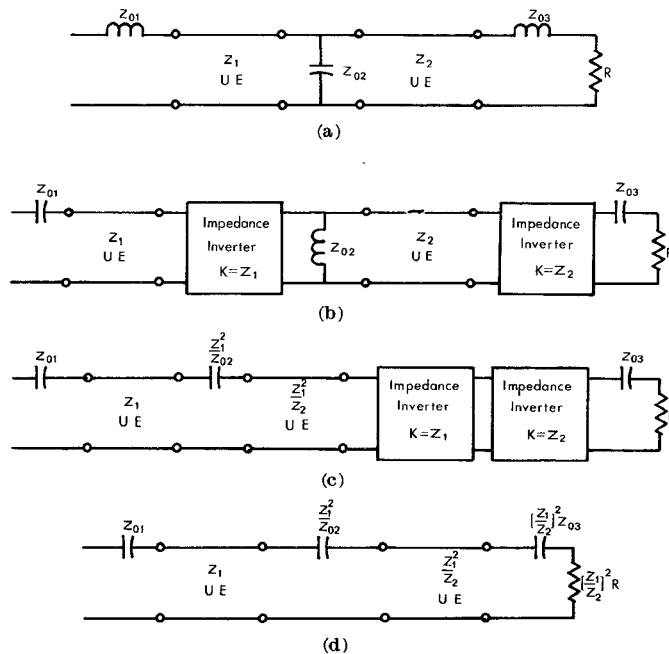

Fig. 3.   (a) A transmission-line network composed of inductive and capacitive stubs and two unit elements. (b) The transform of (a). (c) An equivalent circuit of (b). (d) An equivalent circuit of the circuit of (b) without impedance inverters.

yields the network of Fig. 2(c). Similarly, starting with the network of Fig. 3(a) whose input impedance is $Z(\lambda)$, we obtain the network of Fig. 3(b) with input impedance $Z(1/\lambda)$. Fig. 3(c) and (d) shows the steps used in eliminating the impedance inverters. The extension of this method to more complicated ladder networks is obvious.

The networks thus obtained are identical to those obtained by Cristal [1]. It should be pointed out, however, that the preceding theorem applies to any $Z_L(\lambda)$ and is not restricted to ladder net-

works composed of stubs and unit elements. If one can synthesize $Z_0{}^2/Z_L(1/\lambda)$, then one obtains the network whose input impedance is $Z(1/\lambda)$ as shown in Fig. 1.

REFERENCES

[1] E. G. Cristal, "A frequency transformation for commensurate transmission-line networks," IEEE Trans. Microwave Theory Tech., vol. MTT-15, pp. 348–357, June 1967.